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The implications of time-varying return on portfolio construction

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- Investors alter their portfolios for three major reasons: their investment objective has shifted, they have a date-certain goal, or their asset return expectations have materially changed.
- These changing asset expectations drive what are known as time-varying portfolios, which use forward-looking asset return expectations as the basis for potential allocation changes through time.
- Time-varying portfolios can benefit long-term investors who are willing to take on model risk to achieve certain financial goals. Specifically, time-varying portfolios can potentially achieve effective risk-adjusted "alpha" and target-return solutions.

Introduction

Certain circumstances could make adjusting one's asset allocation prudent.

The investment process typically begins with identifying an investor's specific investment goal—saving for retirement, providing a bequest, purchasing a home, or some other objective. To the extent the objective changes, it may also be prudent for the underlying asset allocation to change.

Portfolio changes driven by date-certain objectives or shifting investment objectives are relatively common. Another widely used measure for changing asset allocation is time horizon. For example, popular target-date products use age as a proxy for changing risk tolerances. The older the presumed investor is, the lower the remaining human capital and presumed tolerance for more volatile assets. This, in turn, can lead investors to shift their portfolio to include a higher bond allocation.

The third, and perhaps less obvious, potential driver of portfolio change is return expectations. All portfolios are built with either implicit or explicit return expectations. Some investors rely on history, while others use a forward-looking forecast. History or steady-state expectations are unlikely to change once they are established. Forward-looking forecasts that utilize current valuation metrics, on the other hand, could change with some frequency, depending on market conditions. Investors who use forecasting to determine future asset returns are likely to change their portfolio allocations as their return expectations change. This paper will examine how time-varying portfolios can help investors and what it takes for these portfolios to be successful.

Who may want to consider time-varying portfolios?

Static, indexed portfolios have wide appeal. Their simple construction, straightforward implementation, and ease of understanding make them highly practical. From an investment theory perspective, static portfolio allocations are based on the presumption that under the efficient market hypothesis, asset returns are expected to equal their long-term average plus or minus "noise" (Cochrane 1999). Since "noise" is the unpredictable portion of returns, this effectively means that asset-return expectations are fixed at their historical average, but with a wide range of potential outcomes period to period.[1]

Time-varying portfolios, by contrast, use long-term forecasted asset-return expectations based on the presumption that changes in asset returns can be predicted with some ability. Predictability of returns has been an area of intense study by academics over the last three decades, giving rise to what Cochrane (1999) terms the "new facts in finance" (NFF).

Cochrane effectively compares asset-return forecasting with a coin flip and the weather. Under the traditional view, return forecasts are like a coin flip—on each flip, the probabilities are the same. The weather, by contrast, changes over time, with the expected temperature in the summer being quite different than in the winter. NFF suggests that although no one will know exactly what the temperature will be tomorrow, there are "seasons" to stock returns, and our expectations of returns will differ over time based on current conditions. As a result, time-varying predictability does not occur over the short-term and perfectly but rather over the longer-term and directionally.[2]

Thus, time-varying portfolios take on active risk in the form of model-based asset allocation changes relative to a static benchmark. Investors who understand the trade-off between model-based forecast risk and the potential for modest outperformance may want to consider time-varying portfolios.

See Appendix A for a mathematical definition of the difference between static and time-varying portfolios.

1 As a result, applying the constant expected returns in a mean-variance optimization framework (Markowitz, 1952) results in the fact that all portfolios that lay on the mean-variance efficient frontier will be static.

2 NFF finds there is some ability to predict future asset returns. Predictability in this instance does not mean that asset returns lack uncertainty; it means that *expected* returns (and the entire distribution of possible returns) follow some sort of systematic predictable pattern over time. Although precise predictability of *actual* asset returns would require perfect foresight into the future, predictability of expected returns requires only that asset returns don't roll in an entirely random fashion over time. Here the word *expected* is critical, and it should be interpreted in a statistical sense. Expected returns refer to the midpoint of the entire distribution of possible return. A sizable body of academic literature further describes this work, including Campbell (1991), Cochrane, (1992), Wolf (2000), Goyal and Welch (2003), Valkanov (2003), Lewellen (2004), and Campbell and Yogo (2005).

Time-varying portfolio strategies

Given the nature of asset-return predictability—long-term and directional, as opposed to short-term and precise—the range of possible time-varying strategies that can benefit from it is rather limited. For instance, so-called tactical asset allocation strategies require a much higher level of precision in the forecasts as well as much shorter forecast horizons, resulting in a higher frequency of portfolio changes.

In contrast, the time-varying portfolio strategies discussed in this paper do not seek to outperform a benchmark over short periods of time, and the portfolios are not usually changed more than once a year, except for meaningful changes in return expectations triggered by outsized market events.

In this paper we discuss time-varying portfolio strategies based on two distinct objectives: optimized risk-adjusted "alpha" and target return.

For the optimized risk-adjusted "alpha" strategy, the Vanguard Asset Allocation Model* (VAAM) is run once a year to solve for the portfolio with maximum expected risk-adjusted return at that point in time for a given investor risk profile. At each point, the VAAM uses the latest five-year asset return forecasts generated by the Vanguard Capital Markets Model® (VCMM). The investor risk profile, in turn, determines the static benchmark portfolio against which the time-varying strategy will be assessed.

For the target-return strategy, the VAAM is also run once a year. However, in this case the objective is for the VAAM to find the portfolio with maximum expected risk-adjusted return within a narrow range of the investor's return target.

Methodology and assessment of the strategies

To assess the benefits and limitations of these two strategies, we conducted a performance analysis using both historical data (the "in-sample" analysis) and real-time VCMM-based return forecasts (the "out-of-sample" analysis). We started with the historical data analysis (annual returns for 1926–2018) to address the proof of concept and identify key modeling choices.

1. Risk-adjusted "alpha" portfolio analysis

Figure 1 illustrates a process chart detailing the methodology for the risk-adjusted "alpha" portfolios in a sequence of steps.

Figure 1. How to construct risk-adjusted "alpha" portfolios using time-varying analysis

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Define the investor's risk profile by determining the appropriate static benchmarks (e.g., 60% stock/40% bond benchmark) and extract the implied risk aversion.	Gather VCMM asset return forecasts for each asset class and for various forecast horizons (e.g., 1-year, 5-year, and 10-year forecasts).	Set a risk budget around the static benchmark portfolio (e.g., a range of +/– 2 percentage points around the benchmark portfolio volatility).	Set any other additional constraints based on business considerations or individual situations (e.g., maximum domestic allocation limited to average home bias in the market).	At each point in time, using VAAM, solve for the portfolio with the best risk-adjusted expected return for the risk aversion determined in Step 1.	Repeat Step 5 once a year or at predefined market events (e.g., major market shift).

Source: Vanguard.

IMPORTANT:

The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM are derived from 10,000 simulations for each modeled asset class. Simulations are from September 1998 through September 2018. Results from the model may vary with each use and over time. For more information, please see "About the Vanguard Capital Markets Model" on page 13.

* Patent pending.

In-sample risk-adjusted "alpha" analysis

For the in-sample risk-adjusted "alpha" analysis, we used a 60% stock/40% bond portfolio as the static benchmark, with global stocks and aggregate global bonds as investment options.

Specifically, we compared the two portfolios using three different modeling considerations, three return measures, and two risk measures. We ran the analysis on multiple forecast periods (one, five, and ten years), holding periods (one, five, and ten years), and rolling assessment periods (one, five, and ten years). The forecast period represents the period of time over which the expected returns are anticipated to materialize. The recalibration frequency represents how often portfolio changes might be made based on the forecast. The rolling period represents the time period over which the results are assessed. For each potential combination of these three elements, we measured multiple return and risk metrics. Returns were calculated measuring the median, the interquartile range (IQR) or middle 50th percentile range, and the total range of results. Risk was assessed using the size of the drawdowns and the dispersion of return distributions. A higher median, tighter IQR, and tighter full range would be the optimal outcome.

The result of these calculations is a grid illustrated in **Figure 2** that identifies three return and risk measurements. The results are color-coded, with green representing outcomes that were better for the time-varying portfolio and red representing better results for the static portfolio.

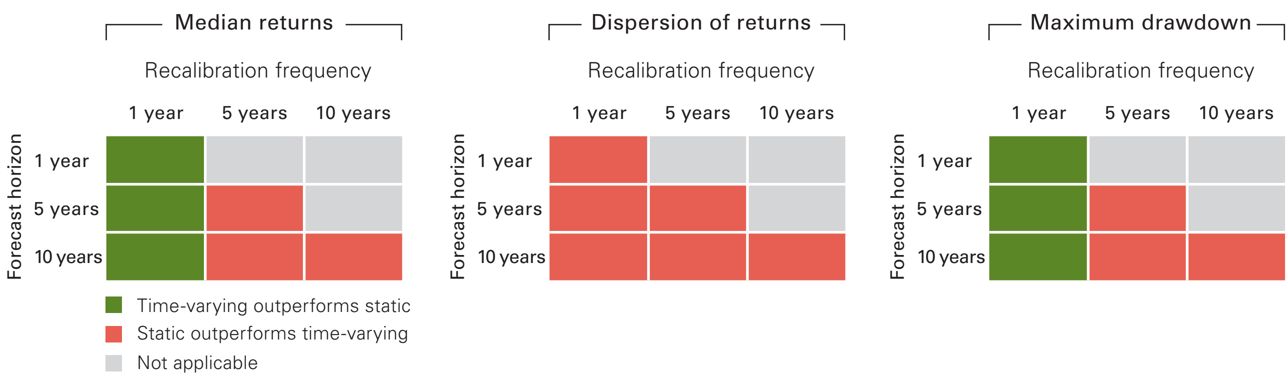
There are three key findings from this in-sample analysis. First, regardless of the forecast period, a one-year recalibration period demonstrated better results for the time-varying portfolios than either the five-year or ten-year recalibration frequencies.

Second, although time-varying portfolios generally produced higher median returns using longer-term forecasts and short recalibration periods, these results come with a wider overall range of potential results. In other words, the in-sample analysis showed that while the median return results were better for time-varying portfolios to achieve these outcomes, an investor would also experience a wider range of overall outcomes. Essentially, there is no "free lunch" if utilizing a time-varying portfolio construction approach—one would need to take on more movement in the portfolio through time to potentially capture better returns.

Third, time-varying portfolios with short recalibration periods produce better downside-risk outcomes, exhibited by lower median and maximum portfolio drawdowns than the static portfolio.

An additional analysis using information ratios (measuring risk-adjusted performance) enables us to better distinguish between the different forecast horizons. Comparing the distribution of information ratios for the one-, five- and ten-year horizons, we find that the five- and ten-year forecast had both a higher median and a tighter IQR. As a result, for the out-of-sample analysis we used the five-year forecast period, with a one-year recalibration period and a five-year rolling assessment period. See **Appendix E** for further details.

Figure 2. Performance outcomes for static and time-varying portfolios (in-sample analysis, 1926–2018)



Note: See Appendix E, Figure E-1, for further details.
Source: Vanguard.

Out-of-sample risk-adjusted "alpha" analysis

The time-varying portfolio produced better results than the static portfolio in the out-of-sample back test on both a performance basis and a downside-risk basis (see **Figure 3**).

Figure 3. Time-varying portfolio outperforms static portfolio (out-of-sample analysis, back-tested results from 1998–2018)

Risk-adjusted "alpha" objective

	Time-varying portfolio	Static portfolio
Median rolling 5-year annualized return	6.6%	6.0%
Maximum drawdown	–17.8%	–21.2%

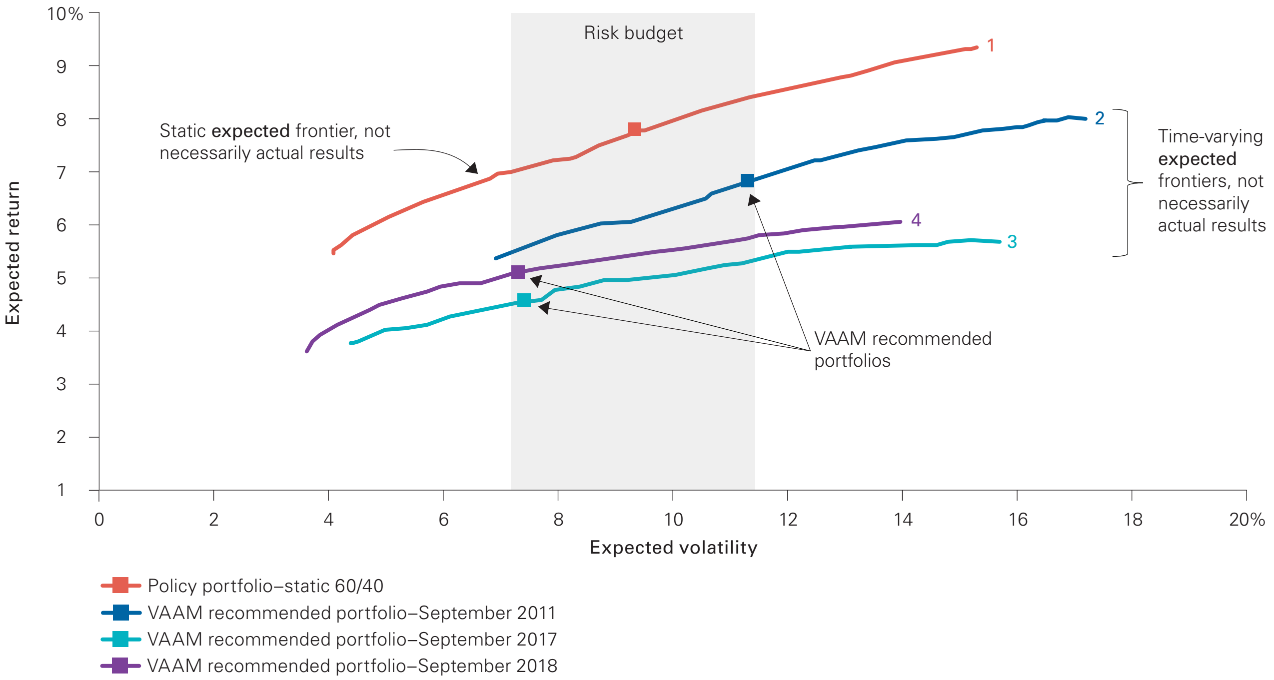
Source: Vanguard.

Please see important information with respect to back-tested information in Appendix D on page 17.

The comparison of a static and time-varying optimized risk-adjusted "alpha" portfolio is illustrated in **Figure 4**. Portfolio allocation decisions were assessed at each year based on the then-calculated forward-looking expectations for the designated forecast period and without the benefit of foresight. As such, the static portfolio was selected using VCMM steady-state (static long-term expectations for risk premia) assumptions in 1998. The subsequent changes to the time-varying portfolio were made (within a risk budget) using the data available in each preceding year, to the extent that expectations changed because of altering market conditions.[3,4]

Figure 4. Hypothetical and back-tested static and time-varying expected frontiers and portfolios (out-of-sample analysis, 1998–2018)

Risk-adjusted "alpha" objective



Note: This chart illustrates a subset of three sample portfolios during the 20-year back-test period to demonstrate the changes that would have occurred.

Source: Vanguard.

Back-tested and hypothetical results are not a guarantee of future returns.

3 Here we utilized a back test from the VCMM to simulate what results would have been through time. Specifically, we simulated a multi-asset time-varying portfolio over 20 years, starting in 1998, to identify both what the through-time asset allocation changes and the aggregate performance calculations would have been. It is worth noting that the value of any back test is to illustrate what practical results could have occurred if the theoretical process in question had been used in the past. At the same time, the results of any back test are dependent on both an array of assumptions and the specific historical time period utilized. Therefore it is reasonable to not place too much emphasis on back-test conclusions as the sole basis for future actions. **Please read important information with respect to back-testing methodology, assumptions, and proxies in Appendix D.**

4 The time-varying portfolios were constrained by three elements: the Vanguard Investment Policy Committee (IPC) advice guidelines, an overall risk budget, and a materiality floor. First, the Vanguard IPC establishes asset allocation ranges within which Vanguard-advised funds typically operate. Second, a risk budget is established to define the maximum change in expected risk the portfolio can take on. Third, a materiality floor is established to set a minimum size over which any signal is required to be. In this case, suggested changes to risk asset exposure need to be at least 5% to be enacted. See **Appendix D** for more details on the static and time-varying portfolio guidelines and constraints.

The figure illustrates a sampling of the expected frontiers (expected outcomes) for the static and time-varying portfolios. The static portfolio (the red box) selected from an efficient frontier (red line, #1) closely mirrors long-term historical returns. For a static 60% stock/40% bond portfolio, the expected return is projected to be 7.8%. By comparison, the time-varying optimized portfolio would change its asset allocation depending on prevailing valuations and future expectations at the time. Three examples of the changing time-varying expected frontiers appear below the original static expectations. This reflects the changing market conditions through the first two decades of the 2000s and the lower expectations of future returns.

Two things are worth noting about the array of frontiers. A change in a portfolio's asset allocation will be driven by the change in the *slope* of the frontier, a measure of the additional return earned for each additional unit of risk. At the same time, a change in the overall level of the frontier will more likely influence the degree to which *expected returns* change. As a result, as expectations change so might the level and slope of returns shift, potentially leading to a change in the time-varying portfolio's asset allocation.

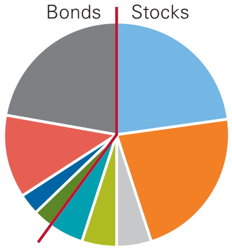
In 2011, the time-varying expectations resulted in a frontier (navy blue line, #2) with a steeper upward slope than the steady state, indicating that investors would be compensated for taking on more risky assets. As a result, the time-varying portfolio is at the top end of the risk budget, maximizing exposure to risky assets. By 2017, expectations changed dramatically enough to result in a flattened expected frontier (teal line, #3), leading to a reduction in risky assets. As a result, the time-varying portfolio was at the lower end of the risk budget. A year later, overall return expectations had improved slightly, but the slope in the frontier had not marginally altered (purple line, #4), resulting in the time-varying portfolio's remaining at the lower end of the risk budget.

The asset class and sub-asset class allocations for the static portfolio and the three time-varying portfolios are detailed in **Figure 5**. The stock allocation for the static portfolio is divided between U.S. stocks (60%) and non-U.S. stocks (40%). The bond allocation includes 70% U.S. bonds and 30% hedged non-U.S. bonds. By comparison, the 2011 time-varying portfolio overweighted risky assets, including equity, value equity factor, and REITs, maintained a neutral weighting to credit bonds, and overweighted international equities relative to U.S. equities. In 2017, the

Figure 5. Asset allocations for static and time-varying portfolios (out-of-sample analysis, 1998–2018)

Risk-adjusted "alpha" objective

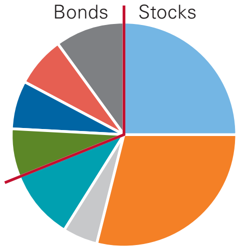
1. Policy
portfolio benchmark



- U.S. equity **23%**
- International equity **22%**
- Value equity **5%**
- Small equity **5%**
- REITs **5%**
- Short-term credit **3%**
- Intermediate-term credit **3%**
- International bonds **12%**
- U.S. Treasury **22%**

60% stock/40% bond portfolio

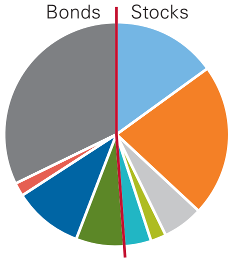
2. September 2011
+9% stocks versus benchmark



- U.S. equity **25%**
- International equity **29%**
- Value equity **5%**
- Small equity **0**
- REITs **10%**
- Short-term credit **7%**
- Intermediate-term credit **7%**
- International bonds **7%**
- U.S. Treasury **10%**

69% stock/31% bond portfolio
- Overweight risk assets
- Overweight REITs
- Underweight U.S. equities
- Underweight international aggregate bonds
- Underweight U.S. Treasuries

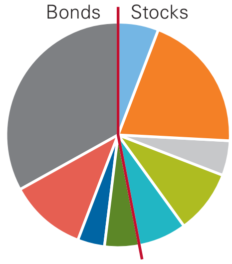
3. September 2017
–11% stocks versus benchmark



- U.S. equity **15%**
- International equity **22%**
- Value equity **6%**
- Small equity **2%**
- REITs **4%**
- Short-term credit **7%**
- Intermediate-term credit **10%**
- International bonds **2%**
- U.S. Treasury **32%**

49% stock/51% bond portfolio
- Underweight risk assets
- Underweight U.S. equities
- Overweight U.S. Treasuries
- Overweight credit bonds
- Underweight international aggregate bonds

4. September 2018
–13% stocks versus benchmark



- U.S. equity **6%**
- International equity **20%**
- Value equity **5%**
- Small equity **9%**
- REITs **7%**
- Short-term credit **5%**
- Intermediate-term credit **4%**
- International bonds **11%**
- U.S. Treasury **33%**

47% stock/53% bond portfolio
- Underweight risk assets
- Overweight REITs
- Underweight U.S. equities
- Overweight U.S. Treasuries

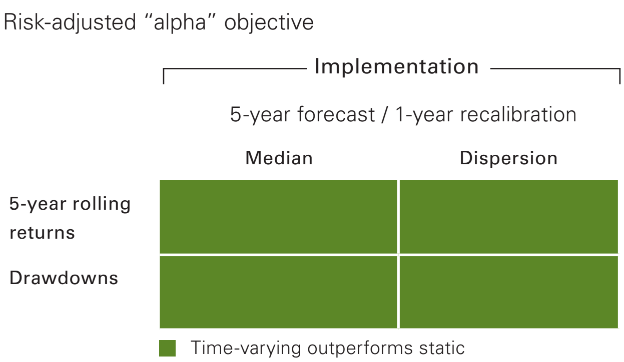
Note: This chart illustrates a subset of four different points in time during the ten-year process to demonstrate the changes that would have occurred.
Source: Vanguard.
Asset allocation does not guarantee a profit or prevent a loss in a declining market.

actions were reversed, and risky assets, in general, were underweighted. A year later, the risky-asset underweight was again increased slightly, maintaining the underweight relative to the benchmark portfolio.[5]

The results of this out-of-sample performance analysis, when using the VCMM forecast in real time over the past 20 years, show that the time-varying portfolio outperformed the static benchmark. Using a five-year rolling assessment period, the median returns were better, while the IQR range and full-range results were tighter. Specifically, the median time-varying return was 6.6%, while the static portfolio return was 6.0%. At the same time, the IQR range for the time-varying portfolio was a tighter 3.0%, compared with the static portfolio range of 3.8%. See **Figure 6** for a summary of the out-of-sample results.

At the same time, from a risk standpoint the time-varying portfolios produced smaller drawdowns relative to the static portfolio. The maximum drawdown for the time-varying portfolio was –17.8%, while the static portfolio's maximum drawdown was –21.2%. See Appendix E for further details on the relative and absolute performance of these scenarios.

Figure 6. Time-varying portfolio outperforms static benchmark (out-of-sample analysis, back-tested results from 1998–2018)



Source: Vanguard.

2. Target-return portfolio analysis

In-sample target-return analysis

Time-varying portfolio construction can also be used to build portfolios that seek a specific level of return. To assess the efficacy of using a time-varying portfolio, we again conducted both an in-sample and out-of-sample analysis. The purpose of the in-sample analysis was to affirm the concept using a broad set of historical data and identify, if possible, the optimal modeling techniques. The purpose of the out-of-sample analysis was to test the concept in real time. **Figure 7** illustrates a process chart detailing the methodology for the target-return portfolio.

For the in-sample analysis, a target return portfolio of 7.0% was established as the objective. Using historical data, we calculated that the given static portfolio to achieve that return was a 37% U.S. stock/63% U.S. bond portfolio over the period from 1926 to 2018.[6]

Figure 7. How to construct target-return portfolios using time-varying analysis

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Determine the appropriate return target and the static benchmark for that return target.	Gather VCMM return forecasts for each asset class and for a 5- or 10-year forecast horizon.	Set a return target range (e.g., +/– 1% around the return target).	Set any other additional constraints based on business considerations or individual situations (e.g., maximum domestic allocation limited to home bias in the market).	At each point in time, using VAAM, solve for the portfolio with the best expected return, taking into account both the return budget parameters and the slope of the frontier.	Repeat Step 5 once a year or at predefined market events (e.g., a major market shift).

Source: Vanguard.

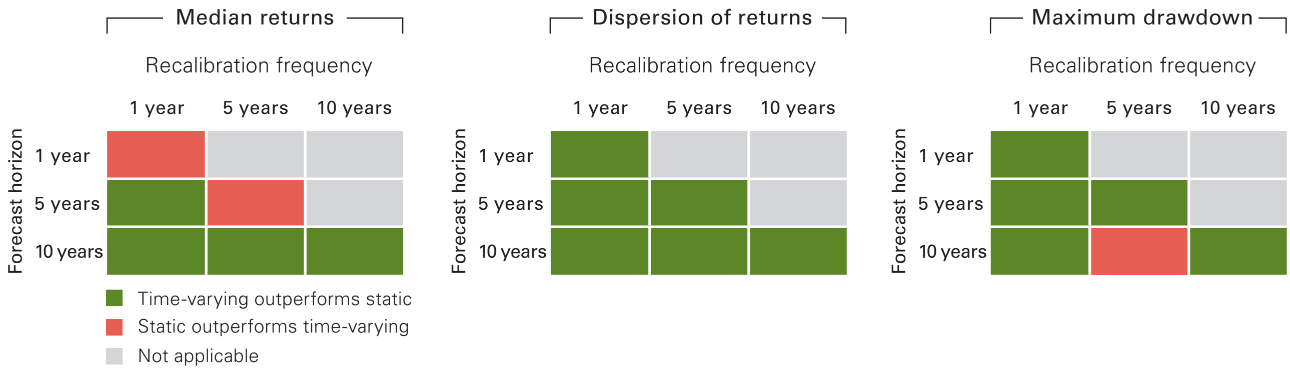
Selecting the time-varying target-return portfolio is a rules-based process defined by a three-step algorithm. The strategy first assesses whether any efficient frontier portfolios have at least 50% probability of achieving the return target. If the answer is no, there is no value in trying to achieve the return target range and the optimization occurs over the entire efficient frontier. This result is observed in portfolios selected in September 2017 and 2018. If the answer is yes, two additional conditions are evaluated. Next, the model assesses whether the least risky portfolio along the efficient frontier has a greater than 50% probability of achieving the return target. If the answer is yes, the entire efficient frontier is above the expected return target, and a 100% bond portfolio is optimized. Lastly, if the answer to the second question is no, the expected efficient frontier captures some or all of the +/– 1% return range around the return target and the asset allocation is optimized within the range. This result is observed in the portfolio selected in September 2011.

5 See **Appendix C** for more details on the portfolio's changing asset allocation over time.

6 The 37% stock/63% bond portfolio was selected over the analysis period from 1901 to 2018. The sub-asset allocation was constructed using 60% domestic/40% international stock and 70% domestic/30% international bonds. Data are sourced from the Dimson Marsh Staunton Global Returns Dataset.

Figure 8. Performance outcomes for static and time-varying portfolios (in-sample analysis, 1926–2018)

Target-return objective



Note: See Appendix E, Figure E-11, for further details.
Source: Vanguard.

The time-varying portfolio was constructed to adjust its asset allocation through time by assessing risk and return trade-offs along the efficient frontier within a +/–1 percentage point return range around the 7.0% return target. We assessed the results over an array of variables looking at one-, five-, and ten-year forecast periods, recalibration frequency, and rolling assessment periods. We compared the two portfolios on both return and risk elements. For return, we calculated the median, the IQR range, and the full range of returns. For risk, we calculated the drawdown and dispersion of return distributions. The results are summarized in **Figure 8**.

Two broad conclusions can be drawn from the chart. First, the time-varying portfolio produced better results than the static portfolio a majority of the time. Indeed, in 15 of the 18 circumstances assessed in Figure 8, the time-varying portfolio produced a median return equal to or better than that of the static portfolio. At the same time, the time-varying portfolios did so while having less severe drawdowns in a majority of the time periods assessed.

Second, the one-year holding period produced better overall results relative to the five- and ten-year holding periods. This indicates that the forecast was most effective when used on an annual basis.

Out-of-sample target-return analysis

The time-varying portfolio produced better results than the static portfolio in the out-of-sample back test on both a performance basis and a downside-risk basis. In addition, the time-varying portfolio more frequently achieved or exceeded the desired target return (see **Figure 9**).

Figure 9. Time-varying portfolio outperforms static portfolio (out-of-sample analysis, back-tested results from 1998–2018)

Target-return objective

	Time-varying portfolio	Static portfolio
Median rolling 5-year annualized return	7.2%	5.9%
Maximum drawdown	–6.4%	–14.0%
Frequency of achieving target return	53.2%	30.6%

Source: Vanguard.

Please see important information with respect to back-tested information in Appendix D on page 17.

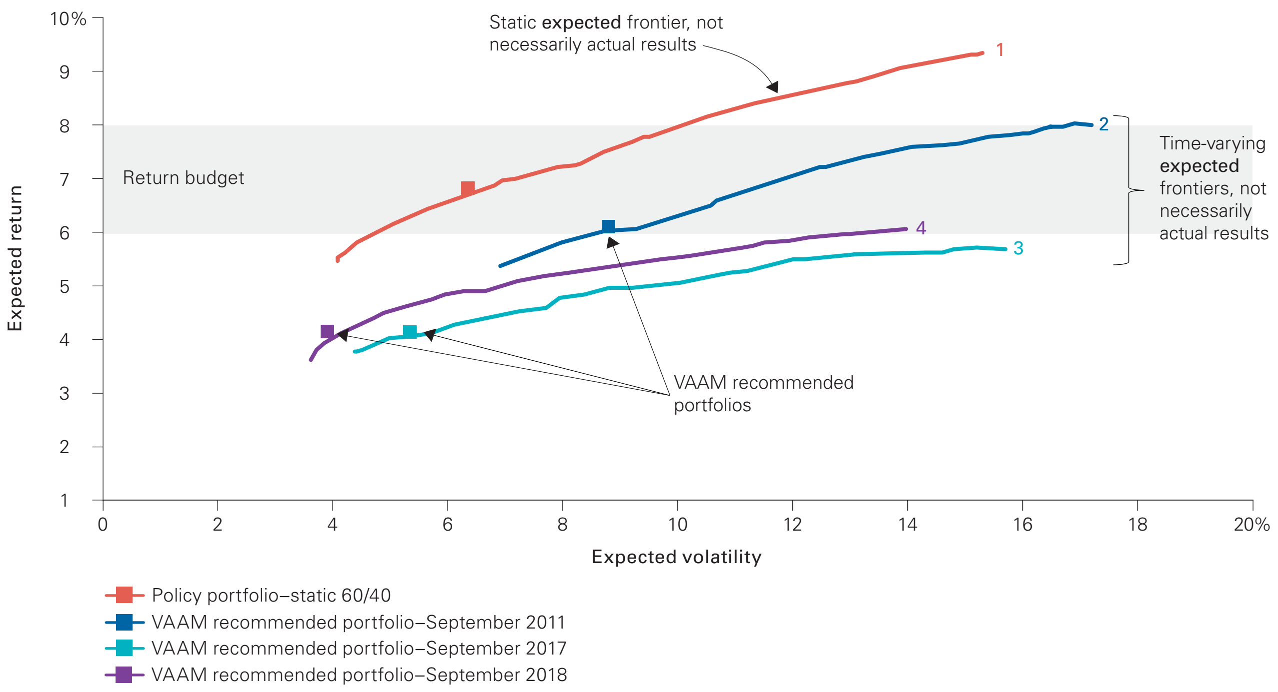
The static portfolio in the out-of-sample analysis was constructed using VCMM steady-state expectations. The resulting portfolio, a 43% equity/57% bond asset allocation, was expected to achieve a 7.0% nominal return.

The static frontier and a sampling of the time-varying frontiers are illustrated in **Figure 10**. The steady-state expectations from which the static portfolio is selected are represented by efficient frontier #1 (the red line). Three sample time-varying sets of expectations are illustrated by efficient frontiers #2 (the navy blue line), #3 (the teal line), and #4 (the purple line). In 2011, return expectations declined, but the slope of the frontier remained steep; as a result, the exposure to equity assets increased. By 2017, the slope of the frontier had both flattened and declined out of the target range of returns.

As a result of these expectations, taking on risk in the portfolio served little purpose, because the expected return difference between the least-risky and most-risky portfolios was modest and none of these results effectively achieved the desired goal. As such, a least-risky portfolio was selected. The situation in 2018 was much the same, with a relatively flat frontier outside of the desired target. Again, a portfolio that eschewed risk was selected. In addition, the 2017 and 2018 circumstances also raise the question of whether the investor should reconsider whether a 7.0% return target is indeed still feasible or should be adjusted downward.

Figure 10. Hypothetical and back-tested static and time-varying expected frontiers and portfolios (out-of-sample analysis, 1998–2018)

Target-return objective



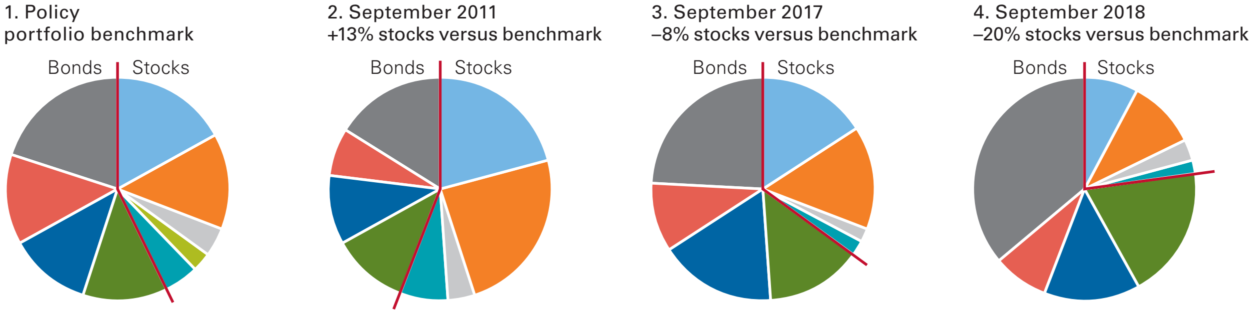
Notes: This chart illustrates a subset of three different points in time during the 20-year back-test period to demonstrate the changes that would have occurred.
Please read important information with respect to back-testing methodology, assumptions, and proxies in Appendix D.
Source: Vanguard.
Back-tested and hypothetical results are not a guarantee of future return.

The details of the static and time-varying asset allocations are illustrated in **Figure 11**. The static portfolio, based on steady-state market expectations, is made up of 43% equities and 57% bonds. International equities made up 14% of the portfolio and international bonds accounted for 13%, for the total 27% international exposure.

By comparison, the time-varying portfolio adjusted its equity allocation widely from 2011 to 2018. In 2011, the time-varying portfolio had a 13% overweight to equities compared with the benchmark portfolio. This shifted to an 8% underweight in 2017, which was expanded further to a 20% underweight in 2018.

Figure 11. Asset allocations for static and time-varying portfolios (out-of-sample analysis, 1998–2018)

Target-return objective



1. Policy portfolio benchmark

- U.S. equity **17%**
- International equity **14%**
- Value equity **4%**
- Small equity **3%**
- REITs **5%**
- Short-term credit **12%**
- Intermediate-term credit **12%**
- International bonds **13%**
- U.S. Treasuries **20%**

43% stock/57% bond portfolio

2. September 2011 (+13% stocks versus benchmark)

- U.S. equity **21%**
- International equity **24%**
- Value equity **4%**
- Small equity **0**
- REITs **7%**
- Short-term credit **11%**
- Intermediate-term credit **10%**
- International bonds **7%**
- U.S. Treasuries **16%**

56% stock/44% bond portfolio
• Overweight risk assets
• Small overweight REITs
• Underweight credit bonds
• Underweight international aggregate bonds
• Underweight U.S. Treasuries

3. September 2017 (–8% stocks versus benchmark)

- U.S. equity **16%**
- International equity **15%**
- Value equity **2%**
- Small equity **0**
- REITs **2%**
- Short-term credit **14%**
- Intermediate-term credit **17%**
- International bonds **10%**
- U.S. Treasuries **24%**

35% stock/65% bond portfolio
• Underweight risk assets
• Underweight REITs
• Small underweight U.S. equity
• Overweight credit bonds
• Underweight international aggregate bonds

4. September 2018 (–20% stocks versus benchmark)

- U.S. equity **8%**
- International equity **10%**
- Value equity **3%**
- Small equity **0**
- REITs **2%**
- Short-term credit **19%**
- Intermediate-term credit **14%**
- International bonds **8%**
- U.S. Treasuries **36%**

23% stock /77% bond portfolio
• Underweight risk assets
• Underweight REITs
• Overweight credit bonds
• Overweight U.S. Treasuries
• Underweight international aggregate bonds

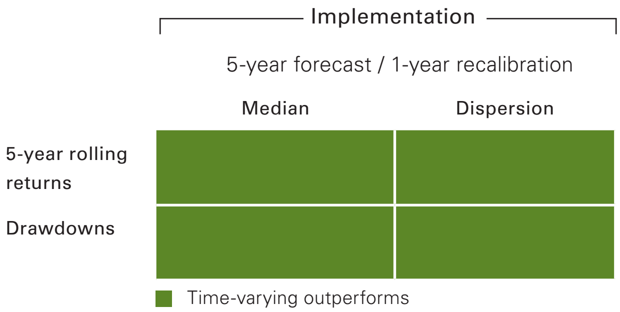
Note: This chart illustrates a subset of four different points in time during the ten-year process to demonstrate the changes that would have occurred.
Source: Vanguard.
Asset allocation does not guarantee a profit or prevent a loss in a declining market.

The out-of-sample performance analysis revealed that, for the circumstances assessed, the time-varying portfolio outperformed the static benchmark. Figure 12 illustrates the cumulative results for the five-year forecast and one-year recalibration period. We found that not only were the median results better for the time-varying portfolio (7.2%, compared with 5.9%) using a five-year forecast and a one-year holding period, but so was the probability that the 7% target would be equaled or bettered more frequently. The time-varying portfolio achieved those results 53% of the time, compared with 31% for the static portfolio. See Appendix E for further details on the relative and absolute performance of these scenarios.

The drawdown analysis found that the time-varying portfolio had similar median results compared with the static portfolio, along with a smaller maximum decline (–6.4%, compared with –14%).

Figure 12. Time-varying portfolio outperforms static benchmark (out-of-sample analysis, back-tested results from 1998–2018)

Target-return objective



Source: Vanguard.

Caveats to using time-varying portfolios

The use of time-varying portfolio solutions comes with three distinct considerations: conviction, patience, and access.[7]

Investors will need to have conviction in the process to act on portfolio construction recommendations that may feel counterintuitive and uncomfortable. All investors face the challenge of handling their emotions in the midst of financial market uncertainty. For a static-portfolio investor, emotion control comes in the form of often needing to stay the course and not take action. But for those using time-varying portfolio strategies, a different type of emotional control is needed—they will need to take action when the model dictates. Here, conviction in the process is needed to combat the fear of a potential misstep. Investors using these models need the resolve to take action when necessary, even if it may feel counterintuitive.

Patience is also called for. All forms of active management require some degree of patience to capture any anticipated benefit. Time-varying strategies use the concepts of NFF, in which future expectations are constructed more like a seasonal weather forecast than predicting a coin flip. As such, it is useful to consider time-varying forecasts as a range of expectations, made directionally, that could take several years to come to fruition. Time-varying forecasts are probabilistic and, as with all probabilistic considerations, often need repeated application to extract value.

Access to a viable tool is another important consideration for using time-varying solutions. For practitioners to implement the NFF concepts in real time, they need access to a working model that can forecast expected returns, volatility, and correlations. This forecasting model acts as the catalyst for portfolio solutions but does not complete the task on its own. It is also important to be able to discern the key trade-offs identified by the model and to implement any solution with tangible investments.

Conclusion

A static, indexed portfolio is a valuable starting point for investors. At the same time, there are conditions under which investors may choose to change their asset allocation. Specifically, if their objective, time horizon, or return expectations change, it may well be prudent to change their asset allocation.

All portfolio solutions incorporate asset expectations, whether implicit or explicit. To the extent the asset expectations are explicit, they could change, depending on the basis that was used to construct them.

Time-varying portfolio strategies, using long-term, forward-looking asset expectations, can form the basis of both risk-adjusted "alpha" and target-return portfolio solutions. In each instance, having a viable model, good judgment, and a tangible investment process are crucial to successfully implementing portfolio solutions that potentially change over time.

7 As with any investment strategy, investors should assess the impact of costs and tax considerations.

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About the Vanguard Capital Markets Model

IMPORTANT: The projections and other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The Vanguard Capital Markets Model® is a proprietary financial simulation tool developed and maintained by Vanguard's primary investment research and advice teams. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the Vanguard Capital Markets Model is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data from as early as 1960. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.

Appendix A

Mathematical equations

Figure A illustrates the core equations and distinct differences for each of three investment strategies.

The basic objective function, return, and risk equations are established at the top of the figure, while the bottom portion illustrates in further detail the key differences of each strategy.

1. Static portfolio

A static total return portfolio seeks to maximize return (Max EU) with no predictive expectation of future returns ($\beta = 0$) and some level of sensitivity to risk ($\lambda > 0$) over a long time horizon.

2. Time-varying risk-adjusted "alpha" portfolio

By contrast, a time-varying total return portfolio, while seeking to maximize the same function (Max EU), has a predictive expectation of future long-term returns ($\beta \neq 0$). Again, there is some sensitivity to risk ($\lambda > 0$), and the horizon is long (X).

3. Time-varying target-return portfolio[8]

Both target-objective strategies utilize an intermediate predictive forecast ($\beta \neq 0$). Target-return portfolios seek a specific level of return, represented as $E(R) = x$. Target-return portfolios can be interpreted as a case of extreme risk-aversion ($\lambda \rightarrow \infty$), where the investor always takes the minimum possible risk, as long as the return target x is attained.

Appendix B

Forecasting time-varying returns

The time-varying nature of asset returns and the existence of predictive signals can, when combined over the long term with a distributive output, help form the rationale for building comprehensive forecasting models. At Vanguard, this has led to the creation of the proprietary Vanguard Capital Markets Model (VCMM), which estimates dynamic statistical relationships between risk factors and asset returns.[9]

The VCMM replaces raw historical averages with alternative forward-looking capital markets steady-state assumptions, so that the simulated variables "revert" toward this equilibrium instead of their historical average values. Simply extrapolating the historical averages, which are highly sample-dependent, might result in misleading returns expectations that could lead to suboptimal portfolio allocations. Regression-based Monte Carlo simulations offer an important advantage in that they can capture current market conditions and the serial correlations of asset returns. This feature shows up in asset-return forecasts that are much more sensitive to changing market conditions, such as changes in stock-valuations multiples or changing interest rates.

Figure A. The mathematical formulas of time-varying portfolios

Portfolio strategies			
Objective function: $EU = E(R_t) - \lambda\sigma(R_t)$ Return: $R_\tau = \mu + \beta X_\tau + \varepsilon_{\tau,}$ Risk: $\sigma(R_\tau) = \alpha + \rho\,\sigma(R_{\tau-1}) + \eta_\tau$			
Item	**Static**	**Risk-adjusted "alpha"**	**Target return**
Portfolio objective and constraint	Max EU	Max EU	Max EU subject to E(R) = x +/– 1%
Return beta Risk beta	$\beta = 0$ $\rho = 0$	$\beta \neq 0$ $\rho = 0$	$\beta \neq 0$ $\rho = 0$
Risk tolerance	$\lambda > 0$	$\lambda > 0$	$\lambda \rightarrow \infty > 0$; subject to Step 5 in Figure 7
Time horizon	Long	Intermediate	Intermediate

Source: Vanguard.

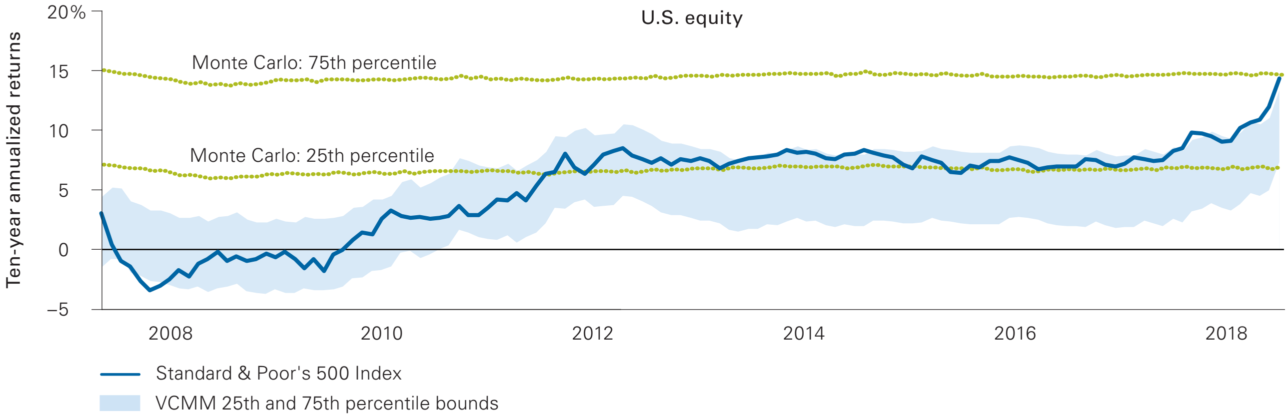
8 A static target-return portfolio would utilize the same objective function as the time-varying target return but with no predictive expectation of future returns ($\beta = 0$).

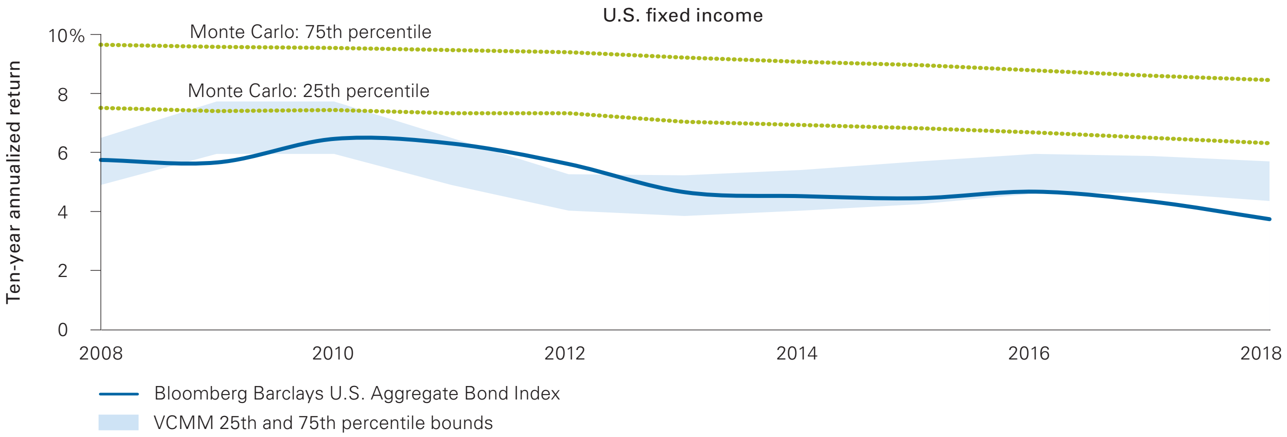
9 The VCMM is a factor-augmented vector autoregression model, although the actual implementation of the VCMM supplements the basic statistical framework with various proprietary, long-run restrictions and different distributional assumptions. For a further discussion on this topic, see *Vanguard Global Capital Markets Model* (Davis et al., 2014).

Figure B illustrates two out-of-sample examples of regression-based Monte Carlo simulations using the VCMM. The change in return forecasts occurs because a dynamic regression-based framework accounts for the initial conditions of various persistent, slow-moving factors—such as the level of interest rates or equity-market valuations. The figure shows the 25th- and 75th-percentile forecasts of ten-year annualized returns. These out-of-sample forecasts are for U.S. equities and U.S. aggregate bond returns beginning in September 2008. For example, as a result, the U.S. aggregate bond December 2008 forecast is a ten-year-ahead forecast that was generated using historical data on or before December 1998.

A key benefit to the VCMM forecast is the distributional nature of the output. Volatility of returns, correlations between various asset-class returns, and correlations between asset-class returns and risk factors are all important. The efficacy of the model is illustrated in Figure B where the actual ten-year annualized returns (dark blue line) fall within the 25th and 75th percentile (light blue bands) VCMM forecasts a majority of the time. Indeed, for the time period covered, the VCMM was successful in more than 87% of the periods for U.S. equity and in more than 63% of the periods for U.S. aggregate bonds. By comparison, independent and identically distributed expected returns (using a similar 25th-to-75th percentile band) produced successful forecasting outcomes 59% of the time for U.S. equity and 0% of the time for U.S. aggregate bonds.

Figure B. Benefits of regression-based Monte Carlo simulations on long-term return forecasts for global bonds and global equity





Notes: Vanguard calculations use back-tested VCMM ten-year annualized forecasts and actual ten-year annualized returns for Standard & Poor's 500 Index and Bloomberg Barclays U.S. Aggregate Bond Index constituents covering periods from September 1998 to November 2018. VCMM forecasts are generated "out of sample" without look-ahead bias by simulating what the current model would have predicted ten years prior using estimation data only up until the forecast is made. Monte Carlo results are simulated using average annual returns and annual volatility based on S&P 500 Index and Bloomberg Barclays data covering December 1976 through September 2018. For additional information with respect to Vanguard's calculation based upon back-tested data, please read important information in Appendix D. Past performance is not a guarantee of future results. Indexes shown are for illustrative purposes only and not representative of any particular investment. Direct investment in an index is not possible.

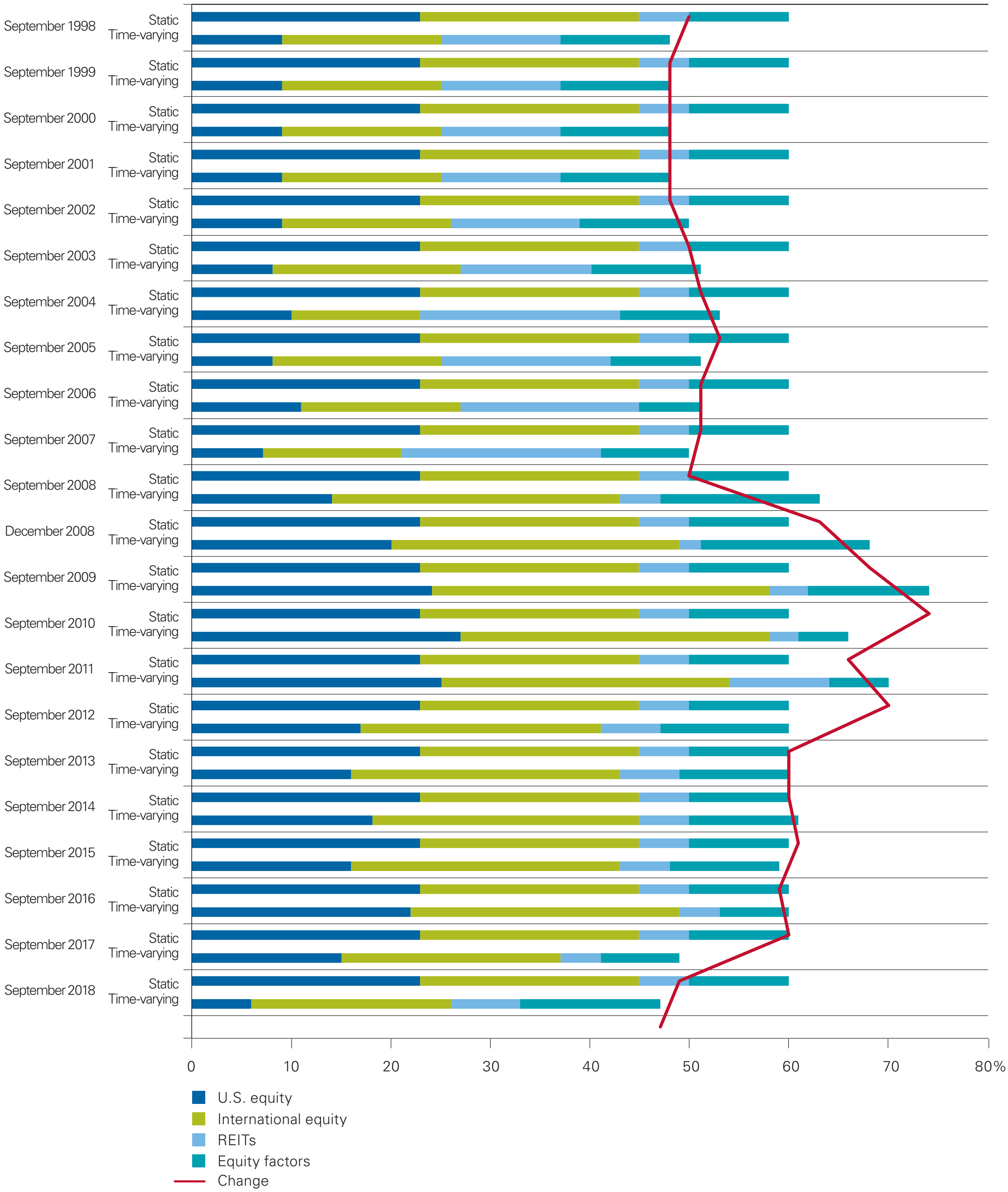
Sources: Vanguard, Barclays Live, and Standard & Poor's.

Hypothetical and back-tested performance is not a guarantee of future results.

Appendix C

Static versus risk-adjusted "alpha" suballocation comparison

Out-of-sample analysis (1998–2018)



Notes: Vanguard calculations are based on back-test results. Time-varying asset allocation changes, represented annually, are based on VCMM forecast output. Time-varying asset allocations are recalibrated based on the September forecast each year, and asset allocation has assumed quarterly rebalancing during the following year. See **Appendix D** for additional back-test disclosure. Data cover September 1998–September 2018. Time-varying asset allocations are recalibrated based on the September forecast each year and additionally at quarter intervals when expected equity risk premium (the difference between expected global equity and global bond returns) changes by +/– 2 percentage points.

Source: Vanguard.

Appendix D: Guardrails for risk-adjusted "alpha" and target-return portfolios

Back-test guardrails

Policy portfolio (benchmark)

- The policy portfolio was created using steady-state return forecasts.

- At the beginning of the back-test period in 1998, asset allocation was created with optimized weights in domestic/international equity, U.S. credit bonds, U.S. Treasury bonds, international aggregate bonds, REITs, and U.S. style factors.

Portfolio constraints

- International equity cannot exceed 50% of total equity.
- International bonds cannot exceed 50% of total bonds.
- REITs cannot exceed 20% of portfolio.
- Aggregate factors exposure cannot exceed 30% of portfolio.
- Credit cannot exceed 50% of total bond allocation.
- Short-term or intermediate-term credit cannot exceed 60% of total credit exposure.

Risk budget: Risk-adjusted "alpha"

The risk budget was set to +/– 2.5% in excess volatility, which equated to +/– 15 percentage points in risk asset exposure difference from the benchmark. Essentially, portfolios could drift between 45% stocks/55% bonds and 75% stocks/25% bonds.

Return budget: Target-return portfolio

The return target was set at 7% with a +/– 1 percentage point difference to establish a range. The portfolio was optimized between that +/– 1 percentage.

Back-test disclosure

To assess the performance implications of implementing time-varying total return solutions, we use the Vanguard Asset Allocation Model (VAAM) to perform portfolio optimizations using capital markets model distribution forecasts in both steady-state (independently and identically distributed) and those considering initial conditions (time-varying). We generated out-of-sample VCMM forecasts without look-ahead bias by estimating the current version of the model only, using sample data up until the time the forecast would have been made in the back-test period. The analysis begins with the selection of the static steady-state portfolio using a mix of global equities and bonds, U.S. REITs, and U.S. small-cap and value equity. Using the VAAM, next we extract the risk preference from its utility function. As expected returns vary over time, so does the implied risk aversion of the steady-state total-return portfolio.

The time-varying portfolio is constructed using the VAAM considering time-varying VCMM forecasts and holding the risk aversion constant. The asset allocation is subject to the constraints detailed here. Time-varying asset allocations are recalibrated annually each September and additionally at quarter intervals if there were significant valuation changes. Asset allocations assumed quarterly rebalancing during the year. Over a 20-year back-test period, quarterly performance is recorded for the time-varying strategy to compare against the static strategic portfolio. The back-tested performance results presented do not reflect any fees or expenses, capital gains, or the reinvestment of dividends. Further, the performance results do not reflect the withholding of taxes. The back-tested performance results are based on criteria applied retroactively with the benefit of hindsight and knowledge of factors that may have positively affected performance, and does not account for all financial risk that may affect actual performance. Accordingly, back-tested results will vary significantly from actual performance.

Proxy disclosure:

U.S. equities: MSCI US Broad Market Index

U.S. short-term credit bonds: Bloomberg Barclays U.S. 1–3 Year Credit Bond Index

U.S. intermediate-term credit bonds: Bloomberg Barclays U.S. 5–10 Year Credit Bond Index

Global ex-U.S. equities: MSCI All Country World ex USA Index

U.S. Real Estate Investment Trusts (REITs): FTSE/NAREIT US Real Estate Index

Global ex-U.S. bonds: Bloomberg Barclays Global Aggregate ex-USD Index

U.S. Treasury bonds: Bloomberg Barclays U.S. Treasury Index

U.S. value-style equities: Russell 1000 Value Index

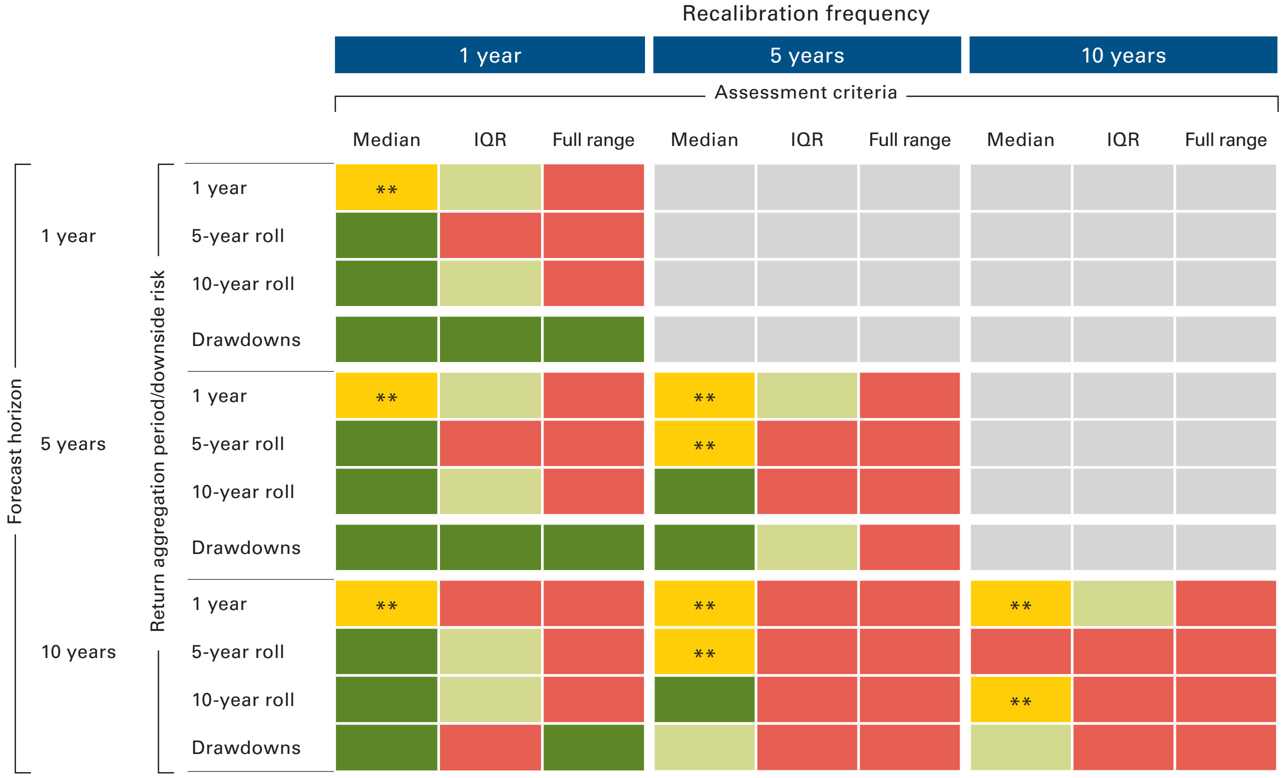
U.S. small-cap equities: Russell 2000 Index

U.S. bonds: Bloomberg Barclays U.S. Aggregate Bond Index

Appendix E

Figure E-1. Risk-adjusted “alpha” portfolio: In-sample analysis

The chart below displays possible assessment combinations measuring the recalibration frequency, forecast horizon, median returns, IQR, full range of returns, and drawdowns.



IQR = Interquartile range (middle 50% of outcomes)
** Time-varying return distributions exhibit positive skewness. Median returns are lower than static median returns, but average returns are higher.

- Improved versus static
- Neutral versus static
- Worse versus static
- Lower median/higher average versus static
- Not applicable

Figure E-2. Risk-adjusted “alpha” portfolio: Distribution of rolling one-year returns (in-sample)

Distribution of one-year returns using a five-year forecast and a one-year recalibration period.

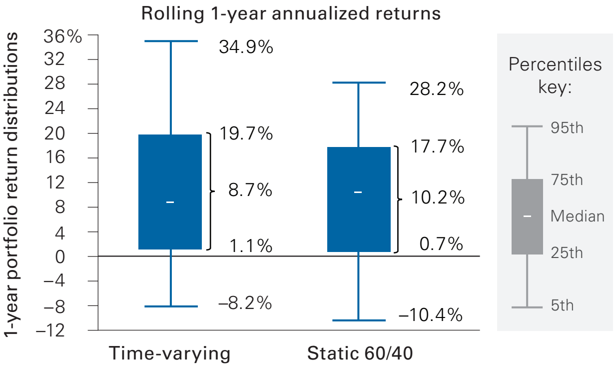


Figure E-4. Risk-adjusted “alpha” portfolio: Distribution of rolling ten-year returns (in-sample)

Distribution of ten-year returns using a five-year forecast and one-year recalibration period.

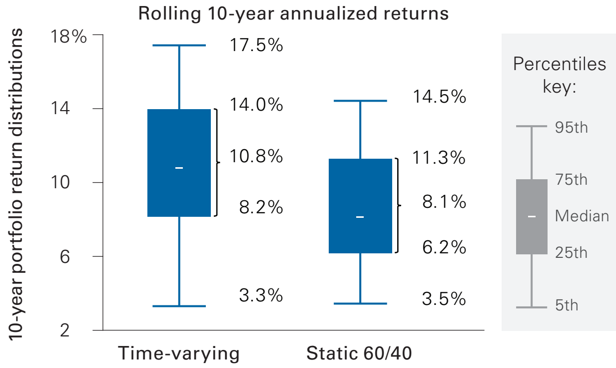


Figure E-3. Risk-adjusted “alpha” portfolio: Distribution of rolling five-year returns (in-sample)

Distribution of five-year returns using a five-year forecast and one-year recalibration period.

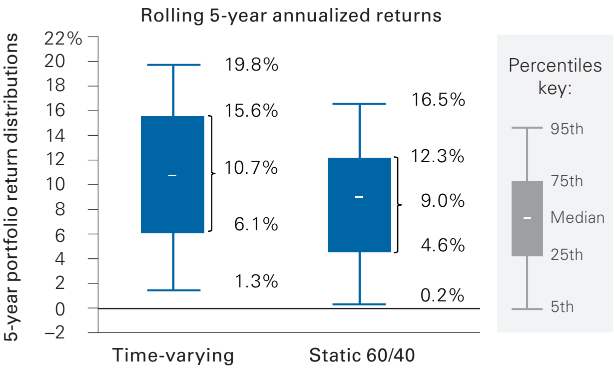


Figure E-5. Risk-adjusted “alpha” portfolio: Distribution of drawdowns (in-sample)

Distribution of drawdowns.

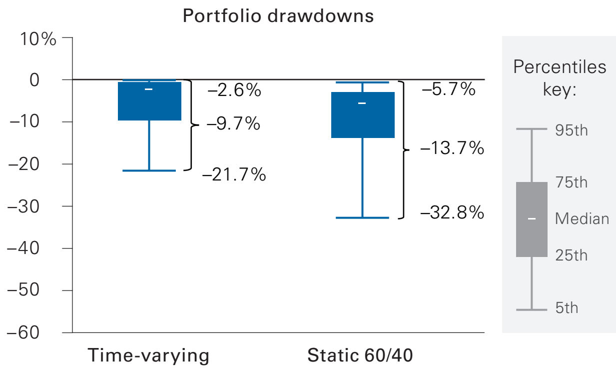


Figure E-6. Risk-adjusted “alpha” portfolio: Distribution of information ratios (in-sample)

Distribution of information ratios for a one-, five-, and ten-year forecast and one-year recalibration frequency.

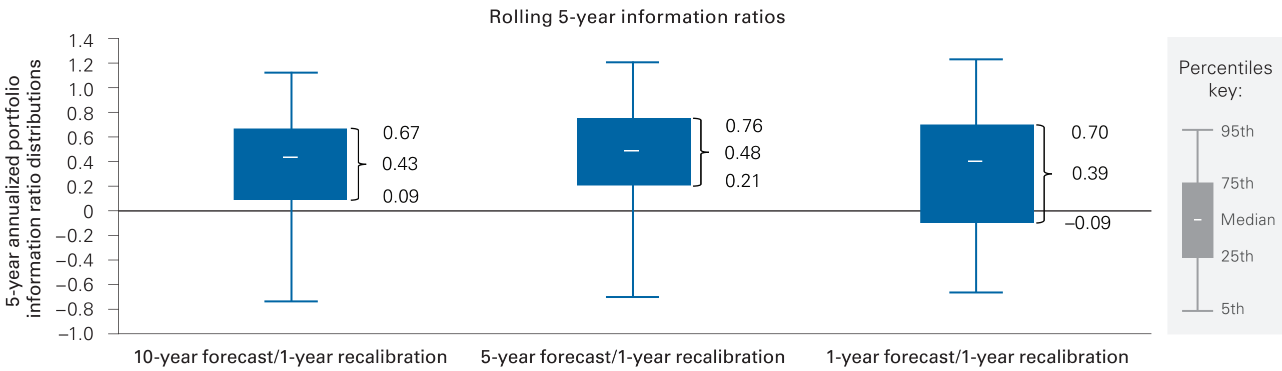


Figure E-7. Risk-adjusted "alpha" portfolio: Distribution of rolling one-year returns (out-of-sample)

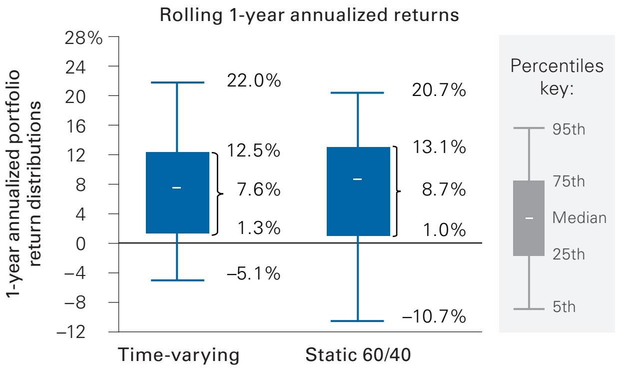


Figure E-9. Risk-adjusted "alpha" portfolio: Distribution of rolling ten-year returns (out-of-sample)

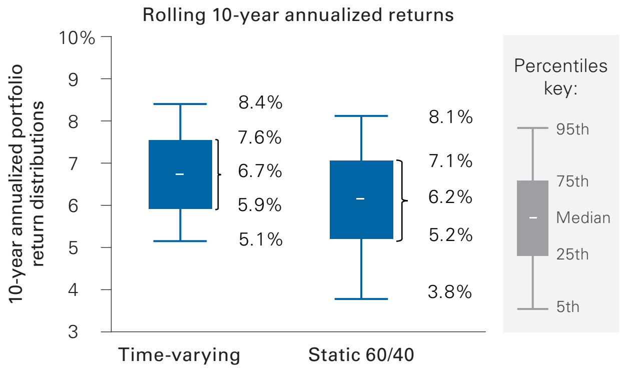


Figure E-8. Risk-adjusted "alpha" portfolio: Distribution of rolling five-year returns (out-of-sample)

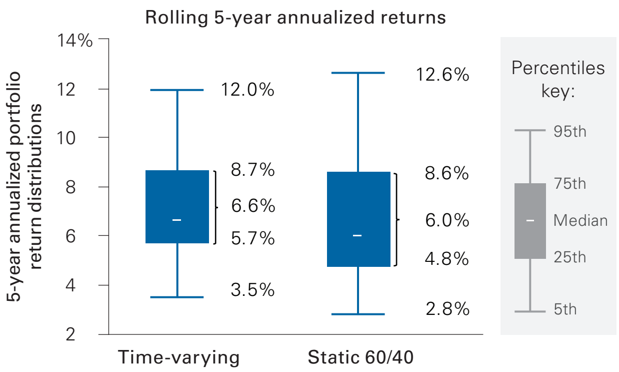


Figure E-10. Risk-adjusted "alpha" portfolio: Distribution of drawdowns (out-of-sample)

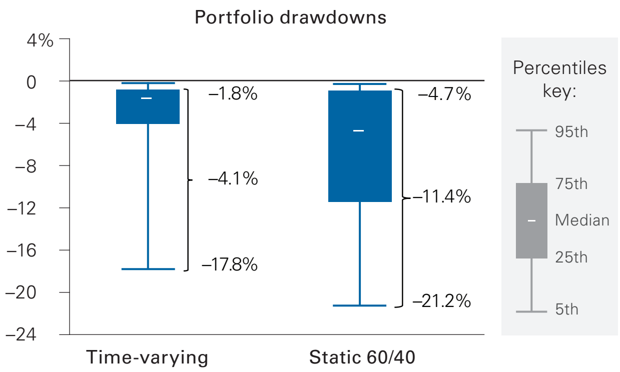
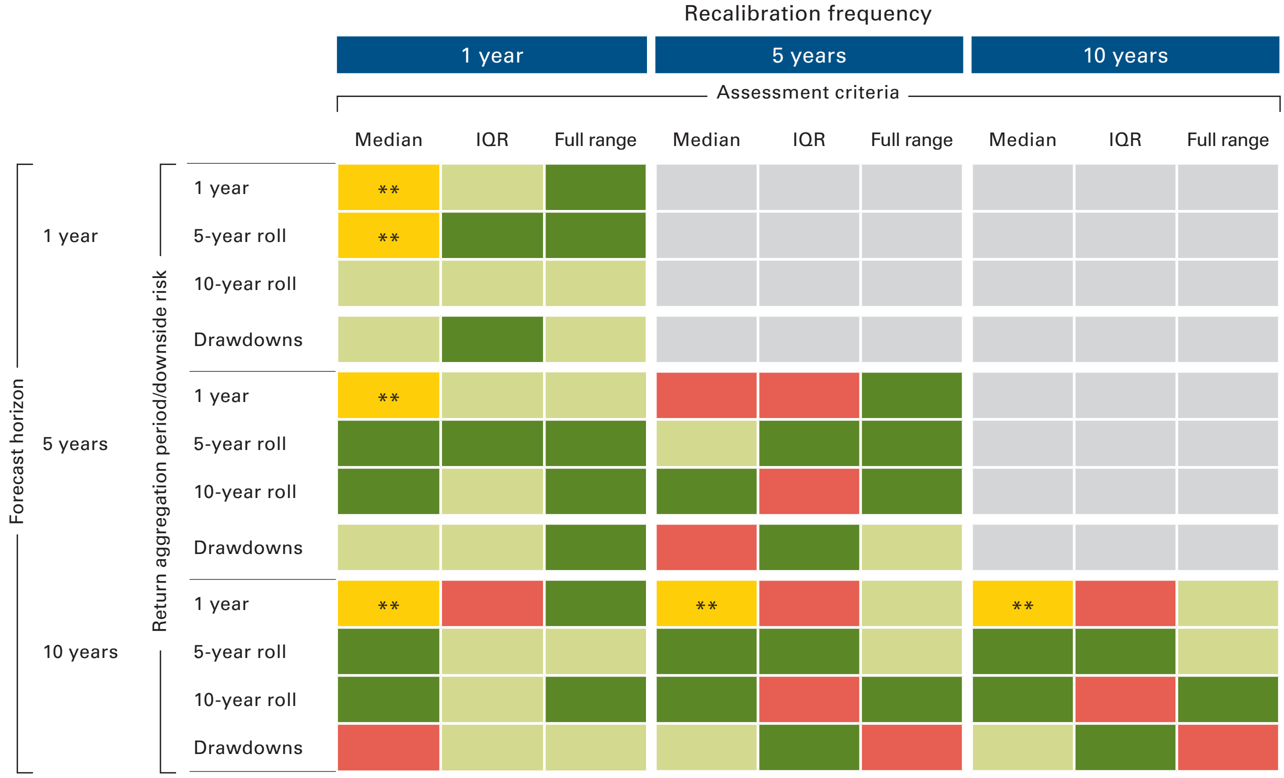


Figure E-11. Target-return portfolio: In-sample analysis

The chart below displays possible assessment combinations measuring the recalibration frequency, forecast horizon, median returns, IQR, full range of returns, and drawdowns.



IQR = Interquartile range (middle 50% of outcomes)
** Time-varying return distributions exhibit positive skewness. Median returns are lower than static median returns, but average returns are higher.

- Improved versus static
- Neutral versus static
- Worse versus static
- Lower median/higher average versus static
- Not applicable

Figure E-12. Target-return portfolio: Distribution of rolling one-year returns (in-sample)

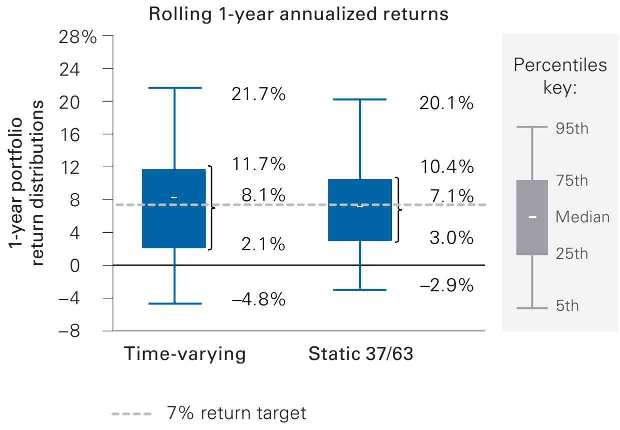


Figure E-14. Target-return portfolio: Distribution of rolling ten-year returns (in-sample)

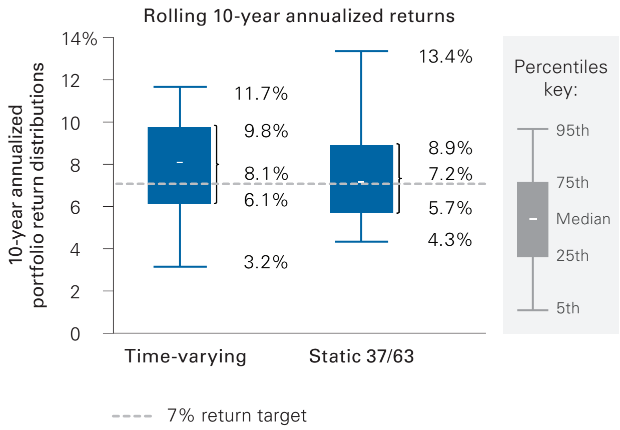


Figure E-13. Target-return portfolio: Distribution of rolling five-year returns (in-sample)

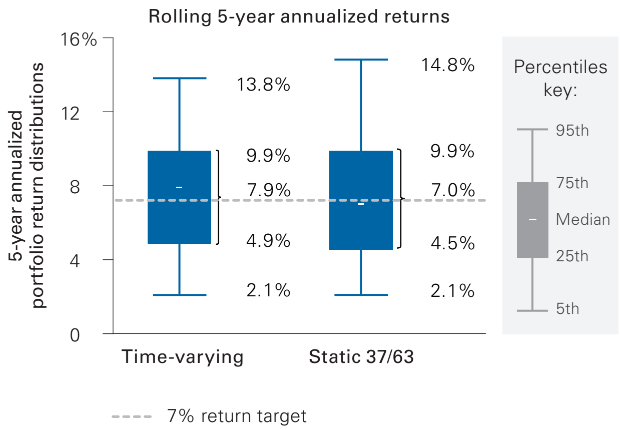


Figure E-15. Target-return portfolio: Distribution of portfolio drawdowns (in-sample)

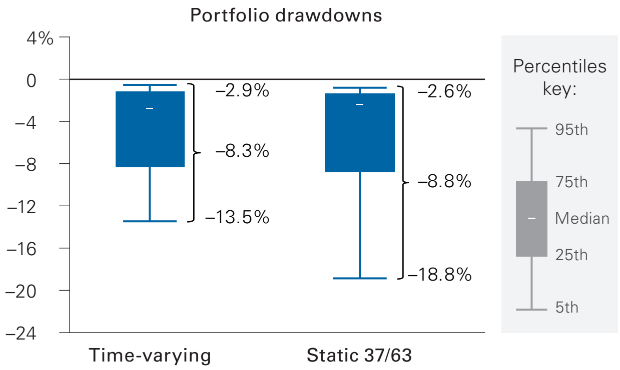


Figure E-16. Target-return portfolio: Distribution of one-year rolling returns (out-of-sample)

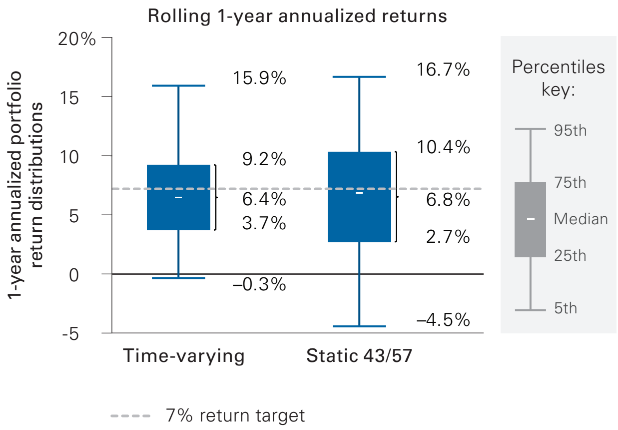


Figure E-17. Target-return portfolio: Distribution of five-year rolling returns (out-of-sample)

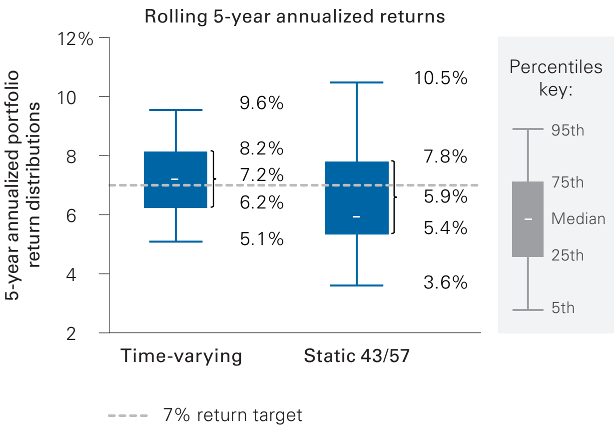


Figure E-18. Target-return portfolio: Distribution of ten-year rolling returns (out-of-sample)

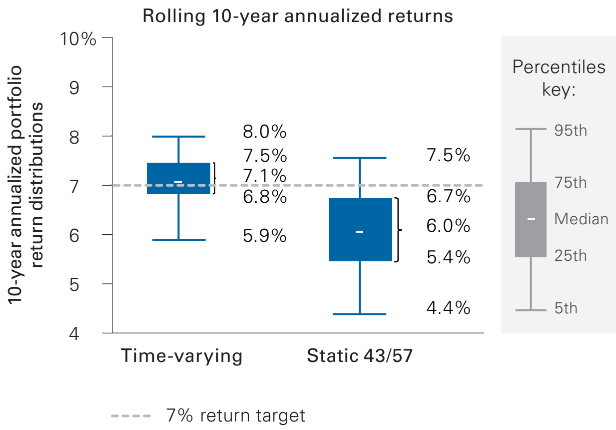
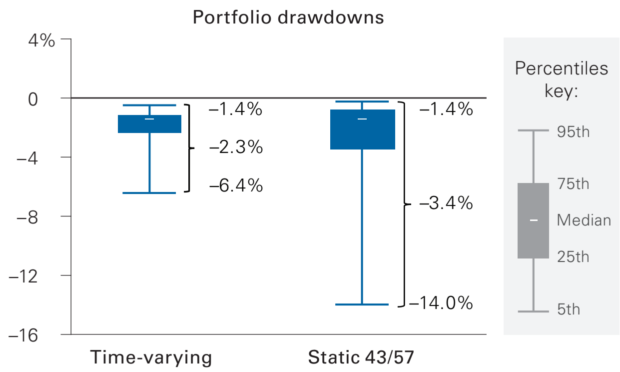


Figure E-19. Target-return portfolio: Distribution of portfolio drawdowns (out-of-sample)



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